J. Fluid Mech. (1967), *vol.* 29, *part* 2, *pp.* 289–295
Printed in Great Britain

The determination of fluctuating velocity in air with heated thin film gauges

By B. J. BELLHOUSE AND D. L. SCHULTZ
Department of Engineering Science, University of Oxford

(Received 26 September 1966)

When attempts are made to use thin film anemometers in airflow to measure fluctuating velocity it is found that the dynamic sensitivity cannot be obtained from a steady-flow calibration. It is found that the dynamic sensitivity is considerably less than that predicted by static calibration and that the sensitivity is frequency-dependent. It is shown that thermal feedback from the substrate, on which the gauge is mounted, to the heated element is responsible for the variation of sensitivity with frequency, despite constant-temperature operation of the probe, and this variation is examined theoretically and experimentally.

1. Introduction

Heated thin film elements mounted in the surface of flat plates, pipes and circular cylinders have been used to measure mean skin-friction by Bellhouse & Schultz (1966). The rapid response of this probe makes the task of distinguishing between laminar, transitional and turbulent flow very simple. In attempts to employ the high-frequency capabilities of thin film probes to measure fluctuating velocity, the authors were struck by the large differences between the measured dynamic sensitivity and that deduced from a static calibration. In efforts to isolate the cause of this discrepancy a series of velocity probes were studied. One such probe, which was wedge-shaped with the thin film deposited along the leading edge, was found to have some advantages. In such a geometry there is no heated substrate upstream of the element, so heat cannot be fed back to the film by forced convection. It would be expected that the steady calibration would apply at all angular frequencies ω much less than U_0/L, the stream velocity divided by the film length in the stream direction. (For $U_0 = 100$ ft./s, $L = 0{\cdot}005$ in., $\omega L/U_0 = 1$, $\omega = 2{\cdot}4 \times 10^5$ rads/s or $f = 38$ kc/s.) The calibrations of two thin film wedges are shown in figure 1. The ordinate is $|(U_1)_s/U_1|$, where $(U_1)_s$ is the fluctuating velocity amplitude measured by the wedge assuming the steady calibration holds instantaneously, and U_1 is the actual fluctuating velocity amplitude. If the steady law holds instantaneously, as it should for the range of $\omega L/U_0$ covered by the experiments, then $|(U_1)_s/U_1|$ should be unity for all frequencies provided no spurious effects are present. In fact the calibrations (figure 1) decrease from unity to approximately 0·5 at about 200 c/s, at which point they remain flat to beyond 1200 c/s, the limit of the calibrating apparatus. The calibration will not deviate from this lower level until the electronic circuits, used to maintain the film at constant temperature, are unable to do so. (The calibration curves supplied by the manufacturer of the constant-temperature equipment (Disa Electronik) were obtained using microwave heating techniques and were above the frequency limit at which thermal feedback would be apparent, and this effect was therefore undetected.)

The probes were calibrated by attaching them to a small plate (1 in. × 1·75 in.) which was oscillated at various frequencies and amplitudes in the streamwise direction by means of an electromagnet. The plate was constrained by taut wires so that movements other than streamwise were inhibited, and its displacement was measured with a capacitance gauge.

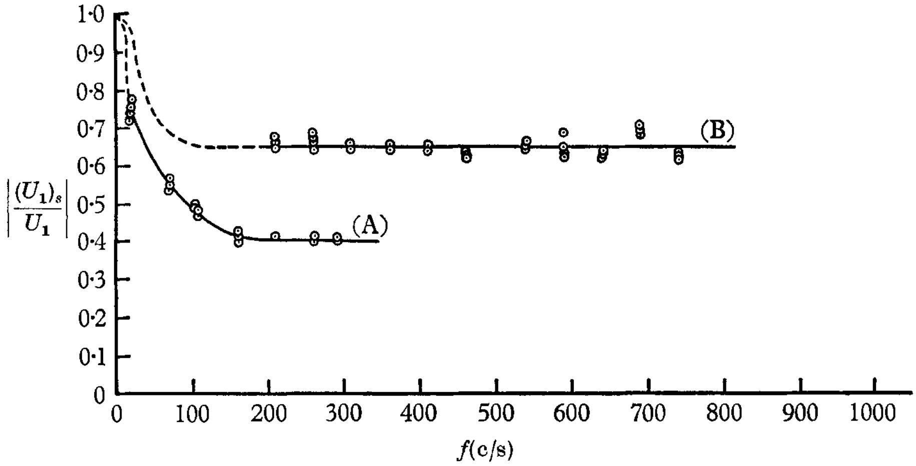


Figure 1. The dynamic calibration of two thin film wedges in air: (A), Oxford thin film wedge; (B), commercial thin film wedge.

Calibrations for thin film probes were similar to those shown in figure 1. It is clear that the quasi-steady calibration gives incorrect results at high frequencies. In order to check the calibration procedure, a hot-wire anemometer was tested, and it did satisfy the quasi-steady law, but difficulty was experienced in producing a probe which did not exhibit a spurious response due to strain-gauge effect caused by the wire oscillation.

The most successful hot-wire probe was made of 0·0005 in. diameter tungsten, which was tensioned between the wire supports. The feedback bridge was able to cope only with frequencies below about 500 c/s with such a substantial wire, and finer wires were not strong enough to stand the tension which was necessary to avoid strain-gauging.

The non-linear relationship between the unsteady heat transfer and frequency is due to heat being conducted through the substrate to the surrounding air, where fluctuating heat transfer takes place, due to the fluctuating air velocity, and thermal waves are returned to the film through the glass. These waves will be sharply attenuated at high frequencies, and so this effect is most noticeable at low frequencies.

2. The effect of thermal feedback on a one-dimensional model of a thin film probe

The usual thin-film probe configuration consists of a small platinum element deposited on the surface of a larger glass substrate, such as the wedge in figure 2(*a*), and heat is given to the glass surrounding the film. The one-dimensional model shown in figure 2(*b*) is simpler to analyse, and it displays the frequency-dependent characteristics found in the first.

The film is maintained at a constant temperature T_w, and heat conduction within the film ensures that T_w is independent of distance, x, and the electronic feedback bridge ensures that T_w is independent of time, t.

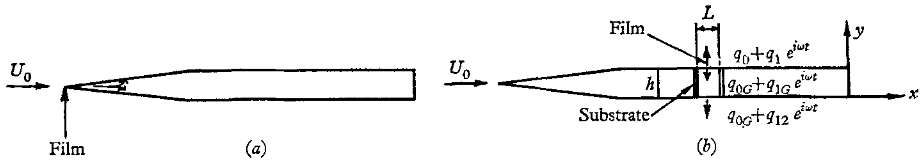


FIGURE 2. The thin film velocity probe and its mathematical model: (*a*) thin film wedge anemometer, (*b*) mathematical model of a thin film anemometer.

The heat-transfer rate from the film directly to the air is $q_0(x)+q_1(x)e^{i\omega t}$, and from the film to the substrate is $q_{0G}+q_{1G}e^{i\omega t}$, and from the lower surface of the element to the air is $q_{0G}+q_{12}e^{i\omega t}$.

Since T_w is independent of x, the temperature gradients within the substrate in the x-direction will be of smaller order than the gradients in the y-direction, provided the thickness of the substrate, h, is not substantially greater than L, the element length.

The heat conduction equation within the glass is

$$\kappa_G\left(\frac{d^2\overline{T}_0}{dy^2}+e^{i\omega t}\frac{d^2\overline{T}_1}{dy^2}\right) = i\omega\overline{T}_1 e^{i\omega t}, \tag{1}$$

where $\overline{T}_0$ and $\overline{T}_1 e^{i\omega t}$ are mean and fluctuating temperatures averaged over the element length, κ_G is the glass diffusivity, and ω the frequency in radians per second.

The steady solution of (1) is

$$\overline{T}_0 - T_w = \frac{\overline{q}_{0G}}{k_G}(y-h), \tag{2}$$

where $\overline{q}_{0G}$ is the steady heat-transfer rate from the film to the substrate, which is of thermal conductivity k_G.

The fluctuating equation is

$$\frac{d^2\overline{T}_1}{dy^2} = \frac{i\omega}{\kappa_G}\overline{T}_1, \tag{3}$$

where the suffix 1 denotes a fluctuating quantity.

At $y = h$, the surface on which the film is deposited, $\overline{T}_1 = 0$. The solution is

$$\overline{T}_1 = \overline{T}_1(0)\left(\frac{e^{(1+i)[\alpha y/h]}}{1-e^{2(1+i)\alpha}}+\frac{e^{-(1+i)[\alpha y/h]}}{1-e^{-2(1+i)\alpha}}\right), \tag{4}$$

where $\alpha \equiv h(\omega/2\kappa_G)^{\frac{1}{2}}$ and $\overline{T}_1(0)$ is the value of $\overline{T}_1$ at $y = 0$. Thus

$$\left.\begin{aligned} \bar{q}_{1G} &= \left(\frac{-2}{e^{(1+i)\alpha}-e^{-(1+i)\alpha}}\right)\frac{k_G\overline{T}_1(0)\,\alpha(1+i)}{h}, \\ \text{and}\qquad \bar{q}_{12} &= \left(\frac{1+e^{2(1+i)\alpha}}{1-e^{2(1+i)\alpha}}\right)\frac{k_G\overline{T}_1(0)\,\alpha(1+i)}{h}. \end{aligned}\right\} \tag{5}$$

When the thickness of the thermal boundary layer produced by a heated element is greater than the viscous boundary layer, Curle (1962) has shown that

$$\frac{d}{dx}\left(\frac{U_0}{q_0(x)}\right) = \frac{\nu q_0(x)}{\beta a_0 k^2 \sigma (T_w - T_f)^2}, \tag{6}$$

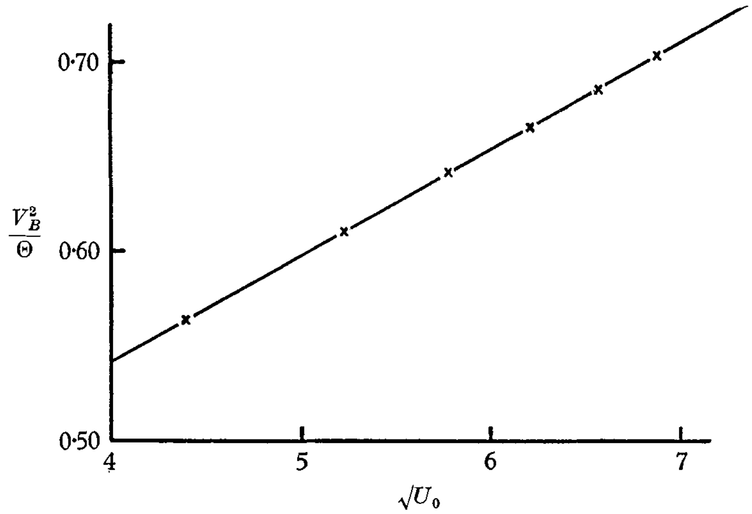


FIGURE 3. Static calibration of a thin film, wedge-shaped, velocity probe.

provided the thermal boundary layer profiles are similar, where

$$(\tfrac{1}{2}\beta a_0)^{\frac{1}{2}} = 0{\cdot}5642 - 0{\cdot}265\sigma^{\frac{1}{2}},$$

ν is the kinematic air viscosity, σ the Prandtl number, k the air thermal conductivity and T_f the air temperature. Integration of (6) yields

$$\frac{\bar{q}_0\, L}{k\Theta} = \left(\frac{2L\beta a_0}{\kappa}\right)^{\frac{1}{2}} \sqrt{U_0}, \tag{7}$$

where $\Theta \equiv T_w - T_f$.

The static calibration of a wedge-shaped velocity probe is presented in figure 3. The ordinate is proportional to the power supplied to the probe, divided by Θ, and the abscissa is $\sqrt{U_0}$. There is a positive value of the ordinate for $\sqrt{U_0} = 0$ which corresponds to the heat loss to the substrate, but the heat given to the air is proportional to $\sqrt{U_0}$ as (7) predicts. Equation (7) will hold instantaneously for small values of the frequency parameter wL/U_0.

Writing $\bar{q}_{1T}$ as the total rate of heat supply to the probe, a heat balance equation for the film takes the form

$$\bar{q}_{1T} = \bar{q}_{1G} + \bar{q}_1. \tag{8}$$

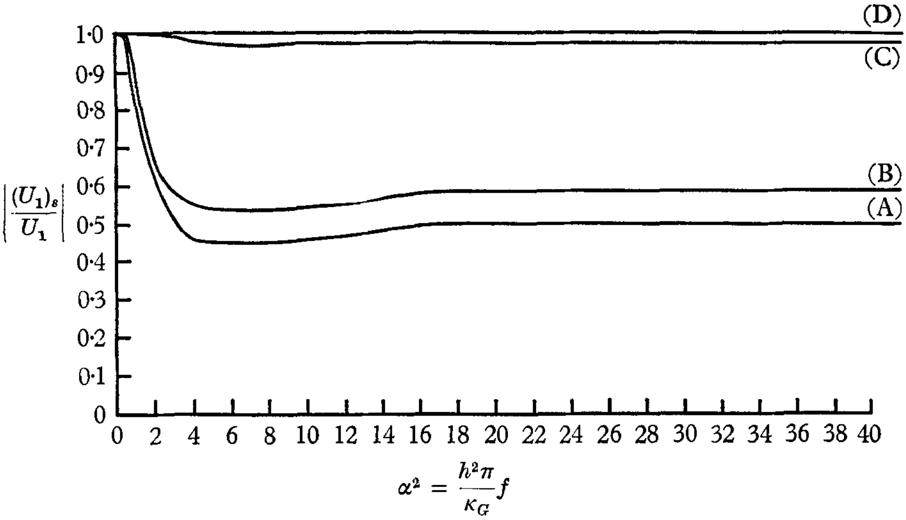
Equation (7) may be applied both at $y = 0$ and $y = h$, and, together with (5) and (8), we obtain

$$\frac{(U_1)_s}{U_1} = \frac{\bar{q}_{1T}}{(\bar{q}_{1T})_{\alpha=0}} = \left(1+\frac{2f_1(\alpha)}{(\gamma\sqrt{U_0}-f_2(\alpha))(1+\gamma\sqrt{U_0})}\right)\left(1+\frac{1}{(1+\gamma\sqrt{U_0})^2}\right)^{-1}, \quad (9)$$

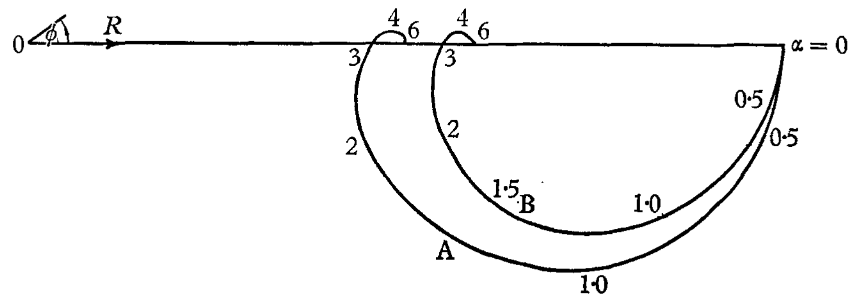
where
$$\gamma = \frac{hk}{Lk_G}\left(\frac{2L\beta a_0}{\kappa}\right)^{\frac{1}{2}},$$
$$f_1(\alpha) = \alpha(1+i)[e^{(1+i)\alpha}-e^{-(1+i)\alpha}]^{-1},$$
$$f_2(\alpha) = \alpha(1+i)[1-e^{2(1+i)\alpha}]^{-1}.$$

It is convenient to write
$$Re^{i\phi} = \frac{(U_1)_s}{U_1} = \frac{\bar{q}_{1T}}{(\bar{q}_{1T})_{\alpha=0}},$$

and values of R and ϕ, deduced from (9), are plotted as functions of α, for various values of $\gamma\sqrt{(U_0)}$, in figures 4(a) and 4(b). It is of particular interest that the theoretical dynamic calibrations for water-flows over 1 ft./s with $h = L$ are



(a)



(b)

FIGURE 4. The calculated effect of thermal feedback on the dynamic calibration of a thin film velocity transducer: (a) $|(U_1)_s/U_1|$ versus f, (b) polar plot, with values of α marked. (A), any fluid at $U_0 = 0$; (B), air at $U_0 = 100$ ft./s; (C), water at $U_0 = 1$ ft./s; (D), any fluid as $\rho c_p k U_0 \to \infty$.

virtually indistinguishable from the quasi-steady, and this is due to the difference in thermal impedance of air and water. Bankoff & Rosler (1962) measured turbulence intensities in air and water with a thin film gauge, and concluded that, while their water measurements were correct, their air measurements were too low by a factor of two or three.

3. Experimental results

A thin film wedge was maintained at constant temperature with a feedback bridge. The current supplied to the film was proportional to the bridge voltage V_B, so the power supplied to the film was proportional to V_B^2. In figure 3 the static calibration is obtained in the form of (7) as a linear relationship between V_B^2/Θ and $\sqrt{U_0}$.

The wedge was shaken in the stream direction, and the true value of the fluctuating velocity was measured with a capacitance gauge.

The dynamic calibrations of two thin film wedges are shown in figure 1, and comparison with figure 4(*a*) shows that the curves are similar, although the theoretical curves apply to the one-dimensional case, and not to the wedge, where heat will be conducted to the substrate surrounding the film. The shape of the tips of the probes, whose calibrations are shown in figure 1, is given by figure 5.

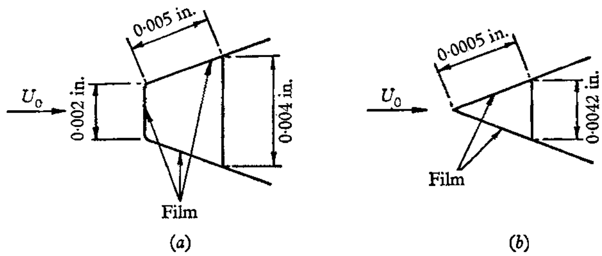


FIGURE 5. Dimensions of the platinum-coated tips of the wedge-shaped velocity probes: (*a*) tip of Oxford probe, (*b*) tip of commercial probe.

The final level of the dynamic calibration may be estimated from the static calibration using the concept of equivalent length. Heat is conducted from the film to the substrate just downstream of the film, and it is convenient to consider a probe of length L_{eff} maintained at a constant temperature T_w. The heat loss from the equivalent probe may be calculated from (7), and compared with the static calibration to obtain the length L_{eff}.

For fluctuating measurements, the effective probe length decreases with frequency (because thermal waves decrease exponentially with the square root of frequency) until at a certain frequency (200 c/s for the probes tested) no heat is fed back to the film from the substrate, and the dynamic effective probe length is precisely the measured length.

The static and dynamic heat-transfer rates measured by means of the bridge voltage will be

$$\int_0^{L_{\text{eff}}} q_0(x)\,dx \quad \text{and} \quad \int_0^{L} q_1(x)\,dx$$

respectively. Thus, if $q(x) \propto x^{-n}$, the final level of the dynamic calibration curve will be $(L/L_{\text{eff}})^{1-n}$. The value of n depends on the geometry of the probe tip, and for the probe whose calibration is given in figure 5 (*b*) the predicted level was 0·5, assuming the tip to be a sharp wedge, while the actual level was found to be 0·41.

It is evidently necessary to calibrate thin film wedges dynamically if they are to be used in air. Perhaps the simplest way is to compare the responses of a thin film and a hot wire to a turbulent air-flow. The method is indirect, and there is the possibility of strain-gauging of the hot wire, but it does avoid the construction of a complicated shaker rig.

For water-flow, the effective length will be close to the measured length, which corresponds to an infinite value of h in the one-dimensional model. Thus the correction for thermal feedback should be negligible for all flow speeds.

4. Conclusion

The effect of thermal feedback from the substrate of a thin film velocity transducer has been shown to be important in air, although it was shown that the effect is independent of frequency above a certain frequency (200 c/s for the probes tested). This effect has been analysed for a simplified model, and the analysis has predicted that no correction is necessary for measurements in water flow.

The authors would like to thank Mr F. H. Bellhouse, who made the probes and calibration rig, and Prof. Turcotte for very helpful discussions.

This work forms part of a programme supported by the Aerodynamics Division, National Physical Laboratory.

REFERENCES

Bankoff, S. G. & Rosler, R. S. 1962 Constant-temperature hot-film anemometer as a tool in liquid turbulence measurements. *Rev. Sci. Instrum.* **33**, 1209–12.

Bellhouse, B. J. & Schultz, D. L. 1966 Determination of mean and dynamic skin friction, separation and transition in low-speed flow with a thin-film heated element. *J. Fluid Mech.* **24**, 379–400.

Curle, N. 1962 *The Laminar Boundary Layer Equations*. Oxford Mathematical Monographs.